

LATTICE BOLTZMANN MODEL FOR MAGNETIC FLUIDS

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Abstract

A lattice Boltzmann model with interacting particles was developed in order to simulate the magneto-rheological characteristics of magnetic fluids. In the frame of this model, $6 + 1$ species of particles are allowed to move across a $2D$ triangular lattice. Among these species, 6 of them carry an individual magnetic dipole moment and interact themselves not only as a result of local collisions, as in current Lattice Boltzmann models, but also as a result of nearest neighbours magnetic dipole-dipole interaction. The relative distribution of the individual magnetic moments is determined by the intensity of an external static magnetic field acting on the whole system.

This model exhibits some relevant characteristics of real magnetic fluids, i.e., anisotropic structure formation as a result of magnetic field induced gas-liquid phase transition and magnetic field dependence of the sound velocity and the attenuation coefficient.

Keywords

magnetic fluids; computational techniques; structure formation

I. INTRODUCTION

Magnetic fluids, also known as ferrofluids, are ultrastable colloidal suspensions of subdomain ferro - or ferrimagnetic particles - e.g., magnetite (Fe_3O_4) - dispersed in various carrier liquids (e.g., water, petroleum, transformer oil, organic solvents, alcohols). These suspensions behave like quasihomogeneous strongly magnetizable liquids due to the presence of approx. $10^{17} - 10^{18}$ magnetic particles in one cubic centimeter and unite the properties of magnetic materials with those of fluids in a rather spectacular way.

Many experimental results suggested that colloidal particles in magnetic fluids always coagulate and form chain clusters, this process being enhanced in the presence of a magnetic field. The formation of chain clusters was observed with an electron microscope [2]. The chain formation process, together with the reorientation of individual particles in the presence of a magnetic field, are responsible for the anisotropy of the physical properties of the magnetic fluids [3]. For example, the transversal magneto-optical effects (birefringence and linear dichroism) induced in thin magnetic fluid layers are well explained by the above-mentioned microstructural processes [4,5]. The sound velocity and the acoustic attenuation coefficient in magnetic fluids are also depending on the angle between the sound propagation direction and the external magnetic field [6].

Several theoretical works [3,7–9] and even computer simulations [10–14] were dedicated to particle interactions in magnetic fluids. The Monte Carlo [10–12], as well as other computer simulations [13,14] performed up to date were limited to magnetic fluids considered only as magnetizable media, but the fluid behaviour of these materials were not considered using such techniques.

The coming out of lattice gas models gave the possibility to introduce particle interactions (other than collisions) in the frame of simple models [15–18] e.g., for immiscible fluids. The recent established Lattice Boltzmann methods having the capability to consider not only the rheological behaviour of multiple components

Lattice gas models, whose first one was the FHP model [24], simulate the exact dynamical history of an integer number of particles moving on a regular lattice while conserving mass and momentum during their collisions. The single-particle equilibrium distribution function specifies the fluid density at each lattice site and also the velocity state, while equilibrium is determined by the particle collision rules. The current trend in cellular automata fluid simulations is to replace the lattice-gas (LG) approach by the Lattice Boltzmann (LB) methods [19–21, 25–28]. A Lattice Boltzmann automaton uses a real-number description for the particle distribution and is far less noisy than the LG approach. It is parallel in nature, due to the fact that all the information transfer is local in time and space, so that it is most suitable for the massively parallel computers.

Following the general LB approach [21], the following lattice Boltzmann equations were considered for a fluid with $S+1$ total components on a two-dimensional ($2D$) hexagonal lattice:

$$n_a^\sigma(\vec{x} + \hat{e}_a, t + 1) - n_a^\sigma(\vec{x}, t) = \Omega_a^\sigma(\vec{x}, t) \quad (1)$$

$$\sigma = 0, 1, \dots, S$$

$$a = 0, 1, \dots, b$$

where $n_a^\sigma(\vec{x}, t)$ is the single particle distribution function for the σ -th component having the velocity directed along the vector \hat{e}_a (see below) and $\Omega_a^\sigma(\vec{x}, t)$ is the collision term. In order to simplify the computer program as much as possible, while retaining the relevant physical aspects, in our investigations we considered that all particles have the same mass, which is set equal to $M^\sigma = 1$ for all $\sigma = 0, 1, \dots, S$.

The particles are located at the nodes of the $2D$ triangular lattice generated by the unit vectors $\vec{e}_1^0 = (1, 0)$ and $\vec{e}_2^0 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ so that any particle position vector \vec{x} is a linear combination of these generating vectors. Since all our investigations were performed on simple geometry $2D$ lattices having almost a rectangular form, the nodes were numbered from 0 to n_{dr} in the X direction

For simplicity, we adopted a single relaxation time form for the collision term and so,

$$\Omega_a^\sigma(\vec{x}, t) = -\frac{1}{\tau} [n_a^\sigma(\vec{x}, t) - n_a^{\sigma,eq}(\vec{x}, t)] \quad (3)$$

where τ is the mean collision time and $n_a^{\sigma,eq}$ is the equilibrium distribution with a given functional form at site \vec{x} and time t . The following form for $n_a^{\sigma,eq}$ is adopted from [19–21]:

$$\begin{aligned} n_a^{\sigma,eq} &= n^\sigma \left[\frac{1-d_0}{b} + \frac{D}{c^2 b} (\hat{e}_a \cdot \vec{u}^\sigma) + \frac{D(D+2)}{2c^4 b} (\hat{e}_a \cdot \vec{u}^\sigma)^2 - \frac{D}{2c^2 b} (\vec{u}^\sigma \cdot \vec{u}^\sigma) \right] \\ n_0^{\sigma,eq} &= n^\sigma \left[d_0 - \frac{1}{c^2} (\vec{u}^\sigma \cdot \vec{u}^\sigma) \right] \end{aligned} \quad (4)$$

where

$$n^\sigma = \sum_{a=0}^{a=b} n_a^\sigma \quad \text{and} \quad \vec{u}^\sigma = \frac{1}{n^\sigma} \sum_{a=0}^{a=b} n_a^\sigma \hat{e}_a \quad (5)$$

are respectively the number density and the averaged particle velocity for the σ -th component at any (\vec{x}, t) after collision and $d_0 \leq 1$ is a constant. Moreover, when equilibrium is reached, one has $\vec{u}^\sigma(\vec{x}, t) = \vec{u}(\vec{x}, t)$ for all $\sigma = 0, \dots, S$ and

$$\vec{u}(\vec{x}, t) \sum_{\sigma=0}^S n^\sigma(\vec{x}, t) = \sum_{\sigma=0}^S \sum_{a=1}^b \hat{e}_a n_a^\sigma(\vec{x}, t) \quad (6)$$

For sufficiently small $|\vec{u}^\sigma|$, the above forms for the equilibrium distribution functions will be positive.

B. Interaction potential

In order to apply the general LB model to magnetic fluids, we consider the particles corresponding to $\sigma = 0$ as being the “carrier liquid” particles, while the “colloidal particles” are those corresponding to $1 \leq \sigma \leq S$. The colloidal particles carry a magnetic moment of magnitude $m = 1$, whose orientation is fixed during the simulation process. For simplicity, we adopted $S = 6$, so that the magnetic moments of each kind of colloidal particles are the vectors:

temperature T , the probability of finding a colloidal particle with a given orientation $\sigma = 1, \dots, S$ becomes proportional to the Boltzmann factor $\exp(-W^\sigma/kT)$, where W^σ is the potential energy of the σ -orientation magnetic particles:

$$\frac{W^\sigma}{kT} = -\frac{\mu_0 \vec{m}^\sigma \cdot \vec{H}}{kT} = -h (\hat{e}_\sigma \cdot \vec{H} / H) \quad (11)$$

with $h = \mu_0 m H / kT$, μ_0 being the magnetic permittivity of the vacuum.

From the above mentioned considerations, one can see that the parameters $\bar{\rho}$, ϕ and h are the main characteristics for any computer run on the basis of our LB model. In order to study the structure formation, we always started our computer runs by assuming that colloidal particles are initially quasi-homogeneously distributed over the lattice with a small 1% random perturbation. Consequently, at $t = 0$, we always had

$$\begin{aligned} n^\sigma(i, j, t = 0) &= \bar{\rho} \phi f^\sigma(\vec{H}) \left(1 + \frac{rand(i, j) - 0.5}{100} \right) \\ &(\sigma = 1, \dots, S) \\ n^0(i, j, t = 0) &= \bar{\rho} - \sum_{\sigma=1}^S n^\sigma(i, j, t = 0) \end{aligned} \quad (12)$$

where $0 \leq rand(i, j) \leq 1$ are uniformly distributed random numbers, $0 \leq i \leq ndx$, $0 \leq j \leq ndy$ and

$$f^\sigma(\vec{H}) = \frac{\exp(\mu_0 \vec{m}^\sigma \cdot \vec{H} / kT)}{\sum_{\sigma=1}^S \exp(\mu_0 \vec{m}^\sigma \cdot \vec{H} / kT)} \quad (13)$$

At the beginning of each run, the magnetic fluid was always considered to be at rest, so that

$$\vec{u}^\sigma(i, j, t = 0) = 0 \quad (14)$$

This implies

$$\begin{aligned} n_a^\sigma(i, j, t = 0) &= \frac{1 - d_0}{b} n^\sigma(i, j, t = 0), \quad a = 1, \dots, b \\ n_0^\sigma(i, j, t = 0) &= d_0 n^\sigma(i, j, t = 0) \end{aligned} \quad (15)$$

During the time evolution, magnetic colloidal particles are supposed to inter-

which becomes simplified if one takes $c = 1$, as mentioned above. Since neutral “carrier liquid” particles do not interact at all, we always have $G_{\sigma\bar{\sigma}a} = 0$ for any $a = 1, \dots, 6$ when $\sigma = 0$ or $\bar{\sigma} = 0$.

Having the interaction potential defined, the rate of net momentum change induced at each site is a simple generalisation of an expression in [21]:

$$\frac{d\vec{u}^\sigma(\vec{x}, t)}{dt} = -n^\sigma(\vec{x}, t) \sum_{\bar{\sigma}=1}^{\bar{\sigma}=S} \sum_{a=1}^{a=b} G_{\sigma\bar{\sigma}a} n^\sigma(\vec{x} + \hat{e}_a, t) \hat{e}_a \quad (19)$$

(the fact that all particles have the mass equal to 1 has been taken into account).

The interaction process is achieved during the collision phase in the LB automaton, i.e., during the collision time $\tau_\sigma \equiv \tau$. The effect of the interaction is to modify the local velocities. Therefore, after the interaction is achieved, the new net momentum $\vec{u}_{new}^\sigma(\vec{x}, t)$ at site \vec{x} for the σ -th component becomes

$$\vec{u}_{new}^\sigma(\vec{x}, t) = \frac{1}{n^\sigma(\vec{x}, t)} \left[n^\sigma(\vec{x}, t) \vec{u}_{old}^\sigma(\vec{x}, t) + \tau \frac{d\vec{u}^\sigma(\vec{x}, t)}{dt} \right] \quad (20)$$

where $\vec{u}_{old}^\sigma(\vec{x}, t)$ is the local velocity before the interaction. In completely uniform equilibrium, there can be no relative flow of particles of different species since these are supposed to have the same mass. Consequently, the particle distribution functions must be locally proportional [30]. Otherwise, different kinds of particles would have different temperature, which is unphysical. For this reason, the local velocity $\vec{u}_{old}^\sigma(\vec{x}, t)$ before the interaction should be chosen always as being the same for all σ , i.e., one has

$$\vec{u}_{old}^\sigma(\vec{x}, t) = \vec{u}(\vec{x}, t) \quad (21)$$

where $\vec{u}(\vec{x}, t)$ was defined by Eq.(6).

The interaction potential does not conserve the net momentum at each site, as usual in current LG and LB methods. However, the total net momentum is conserved on the whole lattice. This can be seen from the symmetry properties of $G_{\sigma\bar{\sigma}a}$:

$$G_{\sigma\bar{\sigma}a} = G_{\bar{\sigma}\sigma a} \quad (22)$$

4. The new equilibrium distribution functions $n_a^\sigma(i, j, t)$ are computed in accordance with Eqs.(4) where $\vec{u}^\sigma(i, j, t) = \vec{u}_{new}^\sigma(i, j, t)$.
5. The Boltzmann equation Eq.(1) and the propagation step are now considered in order to get the propagated distribution functions $n_a^\sigma(\vec{x} + \vec{e}_a, t + 1)$.

D. Conservation laws

The lattice Boltzmann equation

$$n_a^\sigma(\vec{x} + \hat{e}_a, t + 1) - n_a^\sigma(\vec{x}, t) = -\frac{1}{\tau} (n_a^\sigma(\vec{x}, t) - n_a^{\sigma,eq}) \quad (24)$$

can be rewritten after performing a Taylor's expansion ($n_a^\sigma \equiv n_a^\sigma(\vec{x}, t)$; summation from 1 to 2 over repeated greek indices is understood):

$$\partial_t n_a^\sigma + (\hat{e}_a)_\beta \partial_\beta n_a^\sigma + \frac{1}{2} (\hat{e}_a)_\gamma (\hat{e}_a)_\beta \partial_\gamma \partial_\beta n_a^\sigma = -\frac{1}{\tau} (n_a^\sigma - n_a^{\sigma,eq}) \quad (25)$$

Retaining only the first-order derivatives and summing over σ and a , one has [29]:

$$\partial_t \sum_{\sigma,a} n_a^\sigma + \sum_{\sigma,a} \partial_\alpha (\hat{e}_a)_\alpha n_a^\sigma = 0 \quad (26)$$

which, according to Eqs. (6) and (8), is just the continuity (mass) equation:

$$\partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0 \quad (27)$$

The momentum conservation equation is obtained when Eq.(25) is multiplied by $(\hat{e}_a)_\alpha$ and summed over σ and a ; the supplementary term $-\partial_\alpha W$ was added because of the interaction potential:

$$\partial_t (\rho u_\alpha) + \partial_\beta \Pi_{\alpha\beta} + P_\alpha = -\frac{1}{\tau} \sum_{\sigma,a} (\hat{e}_a)_\alpha n_a^{\sigma,neq} - \partial_\alpha W \quad (28)$$

where

$$\rho u_\alpha = \sum_{\sigma,a} (\hat{e}_a)_\alpha n_a^\sigma \quad (29)$$

$$\Pi_{\alpha\beta} = \sum_{\sigma,a} (\hat{e}_a)_\alpha (\hat{e}_a)_\beta n_a^\sigma \quad (30)$$

Since $n_a^{\sigma,eq}$ is an equilibrium distribution function, one has $\partial_t n_a^{\sigma,eq} = 0$ and so,

$$n_a^{\sigma,neq} = -\tau(\hat{e}_a)_\gamma \partial_\gamma n_a^{\sigma,eq} \quad (38)$$

Consequently, we get:

$$\Pi_{\alpha\beta}^{eq} = \frac{c^2}{D} (1 - d_0) \rho \delta_{\alpha\beta} + \sum_{\sigma} n^{\sigma} u_{\alpha}^{\sigma} u_{\beta}^{\sigma} - \frac{\tau c^2}{D+2} [\delta_{\alpha\beta} \partial_\gamma (\rho u_\gamma) + \partial_\alpha (\rho u_\beta) + \partial_\beta (\rho u_\alpha)] \quad (39)$$

Because of the second order derivatives, only $n_a^{\sigma,eq}$ has a relevant contribution to P_α and so,

$$P_\alpha = \frac{c^2}{2(D+2)} \nabla^2 (\rho u_\alpha) \quad (40)$$

The last term in the momentum conservation equation (28) is obtained after a series expansion:

$$\begin{aligned} \partial_\alpha W &= \sum_{\sigma, \bar{\sigma}, a} G_{\sigma\bar{\sigma}a} n_a^{\sigma} [n^{\bar{\sigma}} + (\hat{e}_a)_\beta \partial_\beta n^{\bar{\sigma}}] (\hat{e}_a)_\alpha = \\ &= \sum_{\sigma, \bar{\sigma}, a} G_{\sigma\bar{\sigma}a} n^{\sigma} \partial_\beta n^{\bar{\sigma}} (\hat{e}_a)_\alpha (\hat{e}_a)_\beta \end{aligned} \quad (41)$$

Taking into account the expression (18) of $G_{\sigma\bar{\sigma}a}$, one has

$$\begin{aligned} (M_{\sigma\bar{\sigma}})_{\alpha\beta} &= \sum_a G_{\sigma\bar{\sigma}a} (\hat{e}_a)_\alpha (\hat{e}_a)_\beta = \\ \frac{bc^2}{D} (\vec{n}^\sigma \cdot \vec{n}^{\bar{\sigma}}) \delta_{\alpha\beta} &- \frac{3bc^4}{D(D+2)} [(\vec{n}^\sigma \cdot \vec{n}^{\bar{\sigma}}) \delta_{\alpha\beta} + (\vec{n}^\sigma)_\alpha (\vec{n}^{\bar{\sigma}})_\beta + (\vec{n}^\sigma)_\beta (\vec{n}^{\bar{\sigma}})_\alpha] \end{aligned} \quad (42)$$

where the tensor $(M_{\sigma\bar{\sigma}})_{\alpha\beta}$ is seen to be symmetric

$$(M_{\sigma\bar{\sigma}})_{\alpha\beta} = (M_{\sigma\bar{\sigma}})_{\beta\alpha} \quad (43)$$

Consequently, we have

$$\partial_\alpha W = \sum_{\sigma, \bar{\sigma}} (M_{\sigma\bar{\sigma}})_{\alpha\beta} n^{\sigma} \partial_\beta n^{\bar{\sigma}} = \frac{1}{2} \sum_{\sigma, \bar{\sigma}} (M_{\sigma\bar{\sigma}})_{\alpha\beta} \partial_\beta (n^{\sigma} n^{\bar{\sigma}}) \quad (44)$$

and the x component of the magnetisation

$$M_x(i, j, t) = \sum_{\sigma=1}^S \cos \left[\frac{2\pi(\sigma-1)}{S} \right] \sum_{a=0}^b n_a^\sigma(i, j, t) \quad (46)$$

were retained. The value of the y component of the magnetisation was always found to be very small ($M_y(i, j, t) \simeq 0$) during these computer runs, a fact which is a direct consequence of the x orientation of the magnetic field. The mean value of $M_x(i, j, t)$

$$\bar{M}_x(t) = \langle M_x(t) \rangle = \frac{1}{(ndx+1)(ndy+1)} \sum_{i=0}^{ndx} \sum_{j=0}^{ndy} M_x(i, j, t) \quad (47)$$

was found to be constant during the time evolution of the automaton, as expected.

Figure 1 reproduces the automaton state after $t = 0, 10, 100, 500, 1000$ and 5000 iterations, respectively, for $\bar{\rho} = 0.5$, $\phi = 0.20$ and $h = 0.5$ ($ndx = ndy = 127$). The white points in this figure have $M_x(i, j, t) > \bar{M}_x(t) + 0.1 \cdot \bar{M}_x(t)$, the gray ones have $\bar{M}_x(t) < M_x(i, j, t) \leq \bar{M}_x(t) + 0.1 \cdot \bar{M}_x(t)$, while the black ones have $M_x(i, j, t) \leq \bar{M}_x(t)$. The phase separation, i.e., the onset of thread-like clusters orientated along the magnetic field direction, is evident. Therefore, the white points in Figure 1 belong to the high magnetisation phase, while the black ones in the same figure belong to the low magnetisation phase.

We have computed separately the mean values of the x component of the magnetisation in each phase, i.e.,

$$M_x^{high}(t) = \frac{1}{N_{high}} \sum_{M_x(i,j,t) > \bar{M}_x(t)} M_x(i, j, t) \quad (48)$$

$$M_x^{low}(t) = \frac{1}{N_{low}} \sum_{M_x(i,j,t) \leq \bar{M}_x(t)} M_x(i, j, t) \quad (49)$$

where N_{high} and N_{low} are the total numbers of sites belonging to the high and low magnetisation phase, respectively. A similar procedure was adopted also for the evaluation of the mean densities $\rho^{high}(t)$ and $\rho^{low}(t)$ in the high density and the low density phases, respectively. The resulting values for the run in Figure 1 are reproduced in Table 1.

As the mean values for this run were $\bar{\rho}(t) = 0.5$ and $\bar{M}_x(t) = 0.02426$, one

(AlNiCo8) or type II superconductors, is well evidenced in the frame of this model.

B. Phase transitions

Although the problem of the derivation of an equation of state for our Lattice Boltzmann model was not considered here, we made some attempts (computer experiments) in order to evidence those values of ϕ and h at $\bar{\rho} = 0.5$ which are characteristic for the phase diagram. The results of systematic searches performed up to date are displayed in Figures 3 - 5, where the field dependence of \bar{M}_x , M_x^{high} and M_x^{low} after $t = 5000$ iterations was plotted for $\phi = 0.13$, $\phi = 0.14$ and $\phi = 0.15$, respectively. The upper and lower magnetisation values are initially close to the mean value \bar{M}_x for lower values of the field parameter h . As the field parameter increases, the phase separation is achieved. This process is clearly seen as the bifurcation of the magnetisation curves in Figures 3 - 5. Consequently, when the magnetic field becomes greater than a critical value $h_c \equiv h_c(\phi)$, the high and low magnetisation values become clearly different. The critical field values corresponding to the concentrations in Figures 3 and 4 are $h_c(\phi = 0.13) \simeq 1.1$ and $h_c(\phi = 0.14) \simeq 0.6$. For $\phi = 0.15$, the phase separate even at very low field intensity, $0.0 < h_c(\phi = 0.15) < 0.1$, as one can see in Figure 5.

The separation of magnetic phases at high values of the magnetic field intensity is a characteristic process for magnetic fluids [3,9,14]. This would be inconvenient for many industrial applications, e.g., high speed magnetic fluids rotary seals, but this process is partially overcome by the surfactant layer of the colloidal particles, which always introduces a supplementary repulsive interaction. Therefore, a more realistic Lattice Boltzmann model for magnetic fluids should take this aspect into consideration, in a similar way as in the Monte Carlo models already developed [9–12].

in the mass and momentum conservation laws and get ($\vec{u}'^\sigma = \vec{u}'$), after taking into account that ρ^{eq} is an equilibrium solution and retaining all terms up to the third order :

$$\partial_t \rho' + \rho^{eq} \partial_\alpha u'_\alpha + \partial_\alpha (\rho' u'_\alpha) = 0 \quad (52)$$

$$\begin{aligned} & \rho^{eq} \partial_t u'_\alpha + \partial_t (\rho' \vec{u}'_\alpha) + \frac{c^2}{D} (1 - d_0) \delta_{\alpha\beta} \partial_\beta \rho' + \rho^{eq} \phi \sum_\sigma f^\sigma (\partial_\beta u'_\alpha u'_\beta) - \\ & - \frac{\tau c^2}{D+2} \left[\delta_{\alpha\beta} \rho^{eq} \partial_\beta \partial_\gamma u'_\gamma + \rho^{eq} \partial_\alpha \partial_\beta u'_\beta + \rho^{eq} \partial_\beta \partial_\beta u'_\alpha \right] + \\ & \frac{c^2}{2(D+2)} \rho^{eq} \partial_\beta \partial_\beta u'_\alpha = \\ & - \frac{1}{2} (2\rho^{eq} \partial_\beta \rho' + 2\rho' \partial_\beta \rho') \phi^2 \sum_{\sigma, \bar{\sigma}} f^\sigma f^{\bar{\sigma}} \cdot \\ & \left[\frac{bc^2}{D} (\vec{m}^\sigma \cdot \vec{m}^{\bar{\sigma}}) \delta_{\alpha\beta} - \frac{3bc^4}{D(D+2)} [(\vec{m}^\sigma \cdot \vec{m}^{\bar{\sigma}}) \delta_{\alpha\beta} + (\vec{m}^\sigma)_\alpha (\vec{m}^{\bar{\sigma}})_\beta + (\vec{m}^\sigma)_\beta (\vec{m}^{\bar{\sigma}})_\alpha] \right] \end{aligned} \quad (53)$$

The squared sound velocity $(c_S^1)^2$ is, in the first order approximation, the coefficient of $\delta_{\alpha\beta} (\partial_\beta \rho')$:

$$(c_S^1)^2 = \frac{c^2}{D} (1 - d_0) + \rho^{eq} \phi^2 \left[\frac{bc^2}{D} - \frac{3bc^4}{D(D+2)} \right] \sum_{\sigma, \bar{\sigma}} f^\sigma f^{\bar{\sigma}} (\vec{m}^\sigma \cdot \vec{m}^{\bar{\sigma}}) \quad (54)$$

This result is a generalisation of the squared sound velocity expression in current 2D Lattice Boltzmann models [20,21] and incorporates also the influence of the magnetic field through the distribution functions f^σ , as well as the influence of the colloidal particle concentration ϕ [23,31]

The sound propagation equation is obtained from the conservation equations (52) and (53), subtracting them after their multiplication with ∂_t and ∂_α , respectively, and taking into account the first-order approximation in the continuity equation:

$$\begin{aligned} & \partial_t^2 \rho' - (c_S^1)^2 \nabla^2 \rho' + \frac{(6\tau - 1)c^2}{2(D+2)} \partial_t (\nabla^2 \rho') = \\ & \rho^{eq} \phi \sum_\sigma f^\sigma \partial_\alpha \partial_\beta (u'_\alpha u'_\beta) - \rho^{eq} \phi^2 (\partial_\alpha \partial_\beta \rho') \sum_{\sigma, \bar{\sigma}} f^\sigma f^{\bar{\sigma}} (S_{\sigma\bar{\sigma}})_{\alpha\beta} + \\ & \partial_\alpha (\rho' \partial_\beta \rho') \phi^2 \sum_{\sigma, \bar{\sigma}} f^\sigma f^{\bar{\sigma}} (M_{\sigma\bar{\sigma}})_{\alpha\beta} \end{aligned} \quad (55)$$

where

i.e.,

$$\rho(i, j, t = 0) = \bar{\rho} [1. + \rho_0 \cos(2\pi i/ndx)] \quad (58)$$

Consequently,

$$\begin{aligned} n^\sigma(i, j, t = 0) &= \rho(i, j, t = 0) \phi f^\sigma(\vec{H}) \\ n^0(i, j, t = 0) &= \rho(i, j, t = 0) \cdot (1 - \phi) \end{aligned} \quad (59)$$

The time evolution of the lattice automaton was registered over $n_{iter} = 5000$ iterations, for different values of the concentration ϕ and the field intensity parameter h . The field direction was usually oriented along the x or y axes but, in order to study the anisotropy of sound attenuation, the general case when the angle between the field vector \vec{H} and the x axis is θ , was also considered.

The general behaviour of the space and time dependence of the perturbation (after a mediation over the y direction)

$$\rho'(i, t) = \frac{1}{1 + ndy} \sum_{j=0}^{ndy} [\rho(i, j, t) - \bar{\rho}] \quad (60)$$

was found to be close to the the expression of the attenuated standing waves with the wavenumber $k = 2\pi/ndx$

$$\rho'(x, t) = \rho_0 e^{-\alpha t} \cos(kx) \cos(\omega_S t) \quad (61)$$

i.e.,

$$\rho'(i, t) = \rho_0 e^{-\alpha t} \cos(ki) \cos(\omega_S t) \quad (62)$$

for $i = 0, \dots, ndx$ and $t = 0, \dots, n_{iter}$.

In order to get the interesting quantities α and $\omega_S = kc_S$, which are always accesible to experimental measurements, the x dependence was eliminated ($L = 2\pi/k$ is the lattice length):

$$a(t) = \frac{2}{L} \int_0^L \rho'(x, t) dt = \rho'_0 e^{-\alpha t} \cos(\omega_S t) \quad (63)$$

C. Sound velocity

The typical time evolution of the computed local density perturbation $a(t)$ is reproduced in Figure 6, for $\phi = 0.20$, $h = 0.8$ and two perpendicular orientations of the magnetic field (x and y). The different oscillating frequencies, due to the anisotropy of sound velocity, and also the different attenuation of the sound intensity are very clear. The computed corresponding Fourier spectrum reproduced in Figure 7 also illustrates the sound velocity anisotropy. From these figures, it is very clear that, for the same field intensity, the sound velocity is greater when propagating in the x direction.

On the basis of these first results, a systematic exploration was made in order to see the influence of the magnetic field intensity at a fixed concentration $\phi = 0.20$. The Fourier spectra demonstrated that the sound velocity c_S and consequently, also the angular frequency ω_S , increase in both x and y directions when increasing the value of the parameter h , i.e, when increasing the field intensity while temperature is maintained constant. As mentioned above, the velocity increase in the x direction is always greater than the corresponding increase in the y direction.

Figure 8 shows the concentration dependence of the squared angular frequency ω_s^2 , which was obtained after performing computer runs with the magnetic field $h = 0.8$ in the x direction. One can see that the squared angular frequency is increasing when increasing concentration, a fact which also agrees qualitatively with the theoretical formula (54).

The general behaviour of the sound velocity vs. field intensity, which are retained by our Lattice Boltzmann computer experiments, i.e., the initial increase of the velocity, followed by saturation, agrees well with real experimental measurements e.g., those performed on water based magnetic fluids [32].

D. Sound attenuation

V. CONCLUSIONS

A lattice Boltzmann model with interacting particles was developed in order to simulate the magneto-rheological characteristics of magnetic fluids. In the frame of this model, $6 + 1$ species of particles are allowed to move across a $2D$ triangular lattice. Among these species, 6 of them carry an individual magnetic dipole moment which becomes unchanged during the time evolution of the automaton. These particles interact themselves not only as a result of local collisions, as in usual Lattice Boltzmann models, but also as a result of nearest neighbours magnetic dipole-dipole interaction. The relative distribution of the individual magnetic moments is determined by the intensity of an external static magnetic field acting on the whole system. This model exhibits some relevant characteristics of real magnetic fluids, i.e., structure formation as a result of magnetic field induced gas-liquid phase transition and anisotropy of these structures. The magnetic field induced anisotropy of sound velocity and attenuation in magnetic fluids is also well evidenced in the frame of this model.

The extension of this model in order to allow the particle system to be subjected to time variations of the applied magnetic field amplitude and/or orientation through the introduction of a second relaxation time which may take into account the orientation relaxation of the magnetic colloidal particles, may serve as a basis for the analysis of hot topics related to the magneto - rheological behaviour of magnetic fluids (surface instabilities, pipe flow, magnetic Bénard convection, Taylor vortices formation, heat transfer), as well as an efficient approach to the simulation of the behaviour of some magnetic fluid industrial devices, such as rotary seals, dampers and inductive transducers, onset of the rotary motion of magnetic fluids under the action of rotating magnetic fields and phase transitions induced in magnetic fluids under the action of transient magnetic fields.

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TABLE 1.

t	ρ^{high}	ρ^{low}	M_x^{high}	M_x^{low}
0	0.50000	0.50000	0.02431	0.02418
10	0.50000	0.49999	0.02427	0.02423
100	0.50003	0.49996	0.02435	0.02414
500	0.51433	0.49145	0.05967	0.00147
1000	0.51276	0.49083	0.05440	0.00070
5000	0.51298	0.49083	0.05394	0.00124

TABLE 2.

t	ρ^{high}	ρ^{low}	M_x^{high}	M_x^{low}
0	0.50000	0.50000	0.0060778	0.0060475
10	0.50000	0.50000	0.0060654	0.060599
100	0.50000	0.50000	0.0060635	0.0060619
500	0.500000016	0.499999985	0.0060629	0.0060624
1000	0.500000013	0.499999986	0.0060628	0.0060625
5000	0.500000017	0.499999984	0.0060627	0.0060626

List of figure captions

1. Dynamic evolution of the local magnetisation after $t = 0, 10, 100, 500, 1000$ and 5000 time steps, for $\bar{\rho} = 0.5$, $\phi = 0.20$ and $h = 0.5$.
2. Dynamic evolution of the local magnetisation after $t = 0, 10, 100, 500, 1000$ and 5000 time steps, for $\bar{\rho} = 0.5$, $\phi = 0.05$ and $h = 0.5$.
3. Field parameter (h) dependence of the mean (\bullet), high (\diamond) and low (\square) magnetisation values after $t = 5000$ time steps, for $\bar{\rho} = 0.5$ and $\phi = 0.13$.
4. Field parameter (h) dependence of the mean (\bullet), high (\diamond) and low (\square) magnetisation values after $t = 5000$ time steps, for $\bar{\rho} = 0.5$ and $\phi = 0.14$.
5. Field parameter (h) dependence of the mean (\bullet), high (\diamond) and low (\square) magnetisation values after $t = 5000$ time steps, for $\bar{\rho} = 0.5$ and $\phi = 0.15$.
6. Typical time evolution of the computed local density perturbation $a(t)$ for $\phi = 0.20$, $h = 0.8$ and two orientations of the magnetic field (x and y).
7. Computed Fourier spectrum for the two curves in Figure 6.
8. Concentration dependence of the squared angular frequency ω_S^2 , obtained for $h = 0.8$.
9. Dependence of the attenuation coefficient α vs. the angle θ for $\phi = 0.10$ and $h = 1.0$.
10. Dependence of the attenuation coefficient α vs. the angle θ for $\phi = 0.20$ and $h = 0.5$.